

# Matching Transform model with various 2D polynomial image distortions Paris upgrade



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F.Bonnarel (CDS)



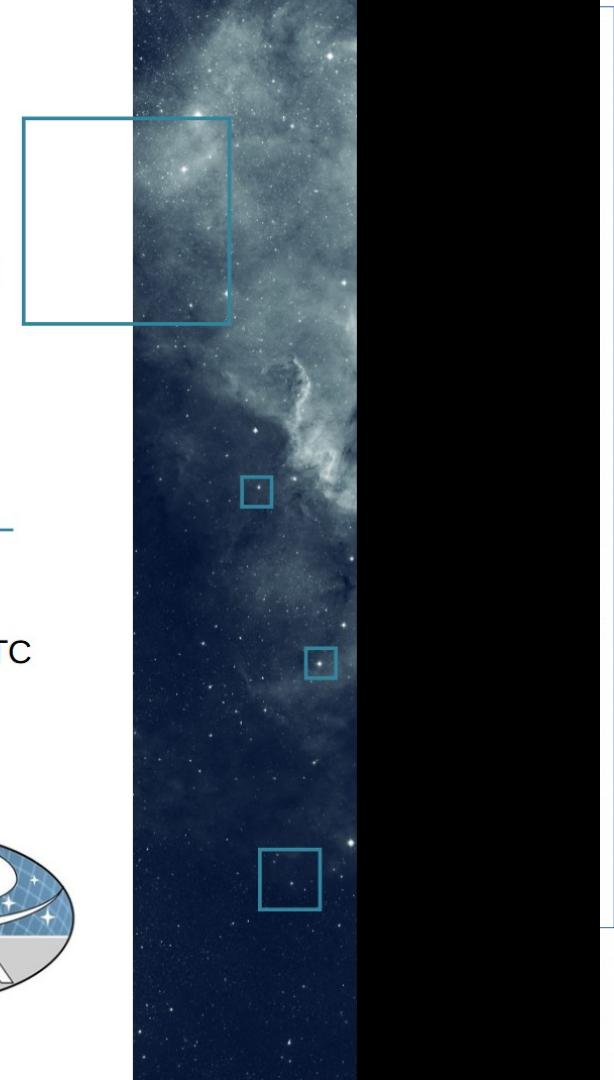
# Follow-up of my College Park talk

Matching STC-2.0 transform model  
with various 2D polynomial image distortions

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F.Bonnarel (CDS)

acknowledges Mark Cresitello-Dittmar for enlightments on the STC  
model and



# Transform datamodel scope

- Data Cubes gather measurement along different axes
  - Sometimes all independant (event lists),
  - Sometimes some are dependant (ND images)
  - « Coordinates » on independant axes.
- Generally pixels are « device » coordinates
- Calibration process allows to map onto World Coordinates
  - Process results in a coordinate transform
- « Transform » allows to represent these coordinate transforms
- Hey !!! Isn't that done via WCS keywords already ?
  - Yes for linear, Not satisfactory for distortions !!!!



# Coordinate transforms : Polynomial distortions

- **Pixels are generally measurement records in the focal plane of the telescope**
- **The linear scheme may be unsufficient to tackle pixel to intermediate coordinates transformation**
- **For large Fields of view the focal plane may become (non plane) focal surface**
- **Introduction of distortions --> 2D Polynomial operations on each pixel coordinate**



# Different methods to code distortions in WCS : failure to standardization (Brian Schmidt, ADASS XXV 2015)

- **SIP coefficients ( $A_{n,m}, B_{n,m}$ ,  $n+m \leq$  polynom order ):** polynomial transformation BEFORE applying bilinear transformation

$X' = A_{0,0} * Dx^0 * Dy^0 + A_{1,0} * Dx^1 * Dy^0 + A_{1,1} * Dx^1 * Dy^1 + ..$

$Y' = B_{0,0} * Dx^0 * Dy^0 + B_{1,0} * Dx^1 * Dy^0 + B_{1,1} * Dx^1 * Dy^1 + ..$

$X = cd1_1 * X' + cd1_2 * Y' \quad Y = cd2_1 * X' + cd2_2 * Y'$

- « TPV » projection code and SCAMP

- usage of  $PVn_m$  parameters.
- Polynomial Transformation AFTER bilinear transformation
- Possible « radial » distortion (skipped below)
- $X' = cd1_1 * Dx + cd1_2 * Dy \quad Y' = cd2_1 * Dx + cd2_2 * Dy$
- $X = PV1_0 + PV1_1 * X' + PV1_2 * Y' + PV1_4 * X'^2 + PV1_5 * X' * Y' + PV1_6 * Y'^2 + ...$
- $Y = PV2_0 + PV2_1 * Y' + PV2_2 * X' + PV1_4 * Y'^2 + PV1_5 * X' * Y' + PV1_6 * X'^2 + ...$



# Different methods to code distortions in WCS : failure to standardization (Brian Schmidt, ADASS XXV 2015)

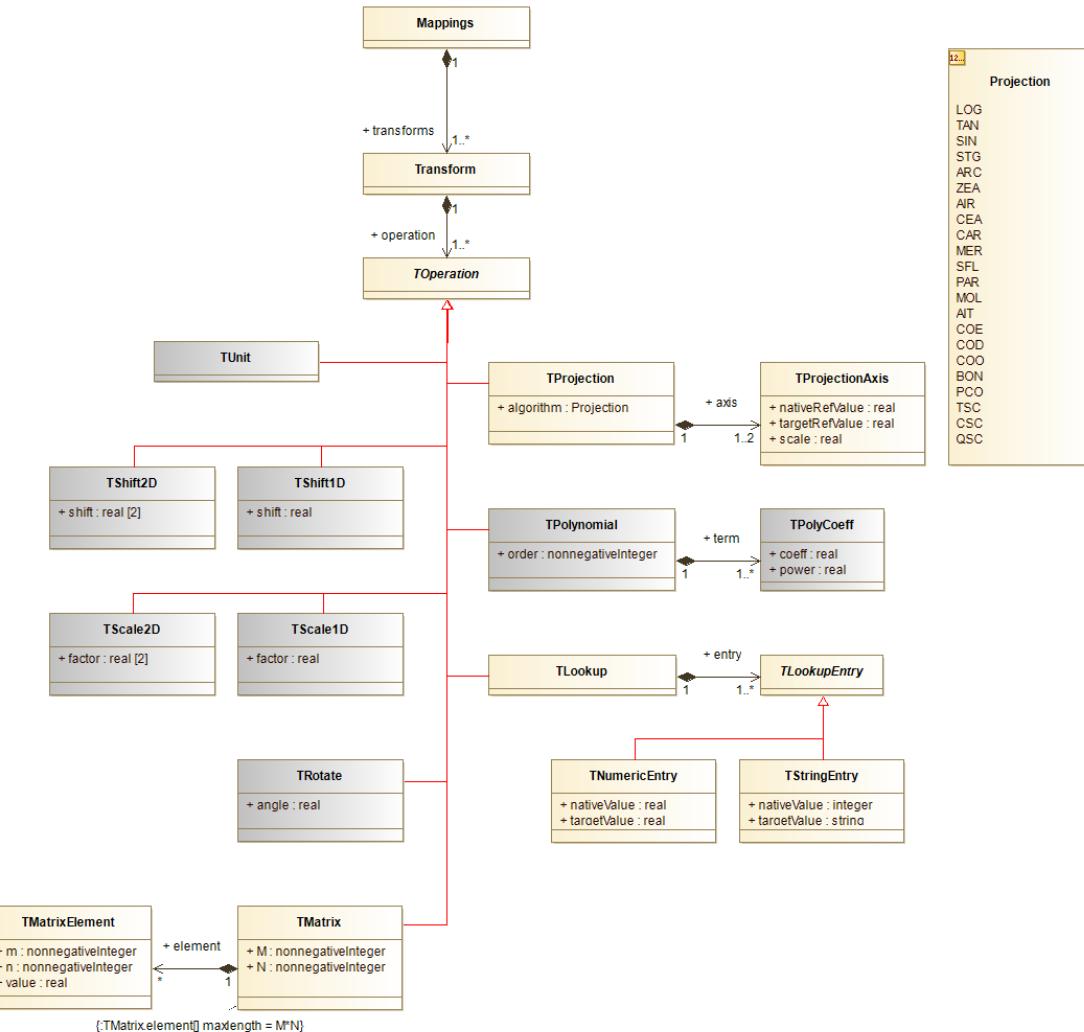
## DSS : no usage of WCS parameters, no explicit BiLINEAR transform

- FITS KeyWORDS : PPO3,PPO6,XPIXELSIZ,YPIXELSIZ, AMDXn,AMDYn...
- $X' \text{ (mm)} = (\text{pp03}-\text{xpixelsiz} * x) / 1000$
- $Y' \text{ (mm)} = (\text{ypixelsiz} * y - \text{pp06}) / 1000$
- $X = \text{amdx1} * X' + \text{amdx2} * Y' + \text{amdx3} + \text{amdx4} * X'^2 + \text{amdx5} * x * y + \dots$
- $Y = \text{amdy1} * Y' + \text{amdy2} * X' + \text{amdy3} + \text{amdy4} * Y'^2 + \text{amdy5} * x * y + \dots$
- $(\text{rad}, \text{dec}) = \text{deProj}(\text{TAN}, X, Y)$

Can Transform datamodel provide an homogenous description for all these « flavors » ?



# STC2 transform model



- Transforms made of successive ordered operations =

- Translations = **Tshift2D**, **Tshift1D**
- Linear transformation = **Tmatrix**
- Polynomial transformation = **Tpolynomial**
- Projection = **Tproj** (**Projection**)
- Scaling = **TScale2D**, **TScale1D**
- Rotating = **TRotate**
- ....

# College Park Conclusion

- Radial distortion to be considered (3D ->2D transform???)
- Extension of Polynomial transform to 2D → 2D (or 3D → 2D) needed
- Apart from that, STC transform provides a unified representation for building transformations by combination of simple operations in any order : Yes !



# New Trans model

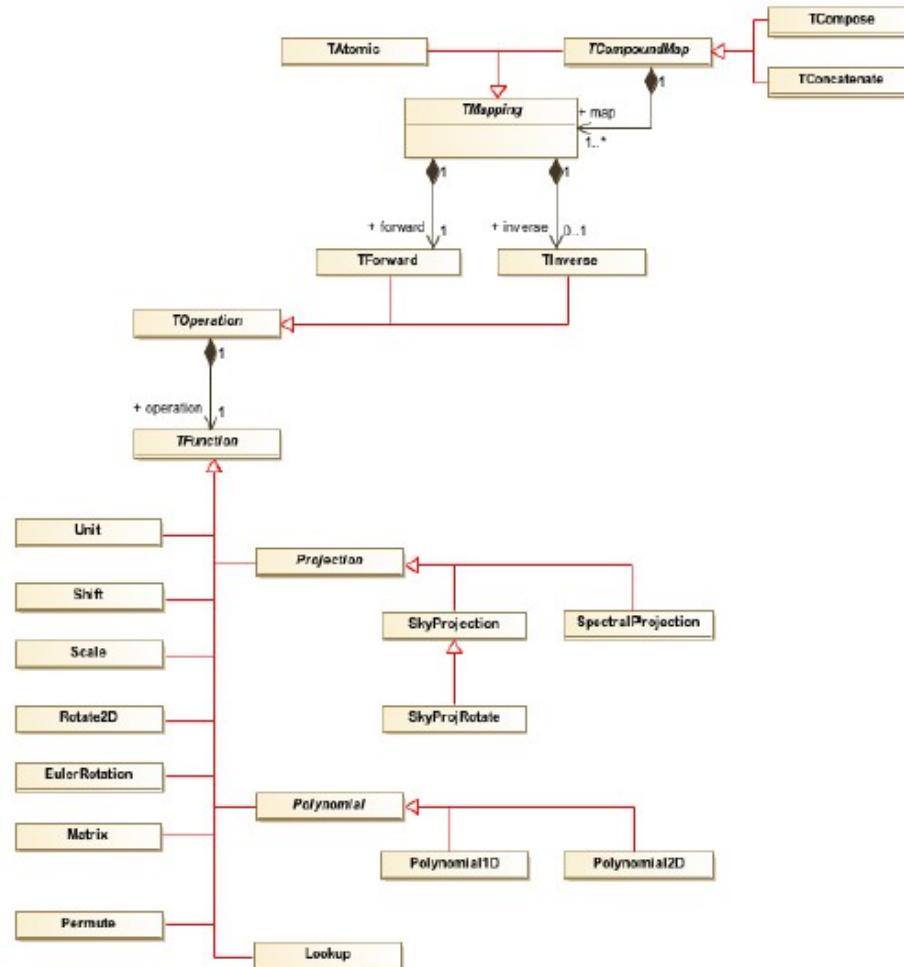


Figure 9: Overview of WCS Transform model elements

# What's new

- **EulerRotation**
- **Polynomial2D**
- **Spectral transformation**
- **Composition (sequence)**
- **Concatenation (parallel)**
- **Invert flag and Tforward/Tinvert operations**



# Simple WCS use case

## STC transform representation (1)

PixelAxis x,y and VirtualAxis to frame « ICRS » and transform made of 3 operations

### 1 Tconcatenate

Tinvert = « true »

- 1 Tforward.Shift.offset = -crpix1
- 2 Tforward.Shift.offset = -crpix2

### 2 Tforward.Matrix

Tinvert = « true »

Tforward.Matrix. M=2

Tforward.Matrix.N=2

Tforward.Matrix.element

Matrix.element.m=1

Matrix.element.n=1

Matrix.element.value=CD1\_1

Tforward.Matrix.element

Matrix.element.m=2

Matrix.element.n=1

.....



# Simple WCS use case

## STC transform representation (2)

### 3 SkyProjRotate

**Tinvert = true**

**Skyprojrotate.algorithm =TAN, SIN, etc.**

**Skyprojrotate.referenceValue[0] = crval1**

**Skyprojrotate.referenceValue[1] = crval2**





# SIP-like WCS STC transform representation (1)

PixelAxis x,y and VirtualAxis to frame « ICRS » and transform made of 4 operations

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## 1 Tconcatenate

**1 Tforward.Shift.offset = -crpix1**

**2 Tforward.Shift.offset = -crpix2**

---



# SIP-like WCS STC transform representation (2)

## 2 TConcatenate

Tforward.Polynomial2D

Polynomial2D.Order = n  
Polynomial2D.term.coeff = A\_2\_0  
Polynomial2D.term.power[0]=2  
Polynomial2D.term.power[1]=0

Polynomial2D.term.coeff = A\_1\_1  
Polynomial2D.term.power[0]=1  
Polynomial2D.term.power[1]=1

.....

Tforward.Polynomial2D

•Polynomial2D.Order = n (may be different than n)  
Polynomial2D.term.coeff = B\_2\_0  
Polynomial2D.term.power[0]=2  
Polynomial2D.term.power[1]=0

Polynomial2D.term.coeff = B\_1\_1  
Polynomial2D.term.power[0]=1  
Polynomial2D.term.power[1]=1

.....

# SIP-like WCS STC transform representation (3)

## 3 ) Tforward.Matrix

Tforward.Matrix. M=2

Tforward.Matrix.N=2

Tforward.Matrix.element

Matrix.element.m=1

Matrix.element.n=1

Matrix.element.value=CD1\_1

Tdorward.Matrix.element

Matrix.element.m=2

Matrix.element.n=1

.....

## 4 SkyProjRotate

Skyprojrotate.algorithm =TAN, SIN, etc.

Skyprojrotate.referenceValue[0] = crval1

Skyprojrotate.referenceValue[1] = crval



# PV-like WCS STC transform representation (1)

**PixelAxis x,y and VirtualAxis to frame « ICRS » and transform made of 4 operations**

-1 Tconcatenate

```
Tforward.shift.offset = -crpix1  
Tforward.shift.offset = -crpix2
```

3 Tforward.Matrix

```
Matrix.M=2  
Matrix.N=2  
Matrix.element  
  m=1  
  n=1  
  value=CD1_1  
Matrix.element  
  m=2  
  n=1  
  .....  
.....
```



# PV-like WCS STC transform representation (2)

## 3 TConcatenate

```
Tforward.Polynomial2D
    Order = n
    Polynomial2D.term
        coeff = PV_1_1
        power[0]=0
        power[1]=0
    Polynomial2D.term
        coeff=PV_1_2
        power[0]=1
        power[1]=0
    .....
Tforward.Polynomial2D
```

```
    Order = n
    Polynomial2D.term
        coeff = PV_2_0
        power[0]=0
        power[1]=0
    Polynomial2D.term
        coeff=PV_2_1
        power[0]=0
        power[1]=1
    .....
```

# PV-like WCS STC transform representation (3)

4

**SkyProjRotate**

**Skyprojrotate.algorithm =TAN, SIN, etc.**

**Skyprojrotate.referenceValue[0] = crval1**

**Skyprojrotate.referenceValue[1] = crval2**



# DSS-like FITS header solution-> STC transform representation (1)

**PixelAxis x,y and VirtualAxis to frame « ICRS » and transform made of 5 operations**

**1 Tconcatenate\$**

Tforward.scale

Scale.factor = xpixelsiz

Tforward.scale

Scale.factor = ypixelsiz

**2 Tconcatenate**

- Tforward.shift

- Shift.offset = -pp03

- Tforward.shift

- Shift.offset = -pp06

**3 Tconcatenate**

Tforward.scale

Scale.factor = -1/1000

Tforward.scale

Scale.factor = 1/1000



# DSS-like FITS header solution-> STC transform representation (2)

## 4 TConcatenate

```
Tforward.Polynomial2D
    Order = 3
    Polynomial2D.term
        coeff = AMDX1
        power[0]=1
        power[1]=0
    Polynomial2D.term
        coeff=AMDX2
        power[0]=0
        power[1]=1
    .....
Tforward.Polynomial2D
    Order = 3
    Polynomial2D.term
        coeff = AMDY1
        power[0]=0
        power[1]=1
    Polynomial2D.term
        coeff=AMDY2
        power[0]=1
        power[1]=0
    .....
```

# DSS-like FITS header solution-> Transform representation (3)

5

## **SkyProjRotate**

**Skyprojrotate.algorithm** =TAN, SIN, etc.

**Skyprojrotate.referenceValue[0]** = crval1

**Skyprojrotate.referenceValue[1]** = crval2



# Possible serialisations

**1 ) json : from Top mapping element to leaves**

--> hierarchy of embedded json objects

Composition : sequence

Concatenation : array

Object names = « vo-dml ids »

**2 ) VODML-lite mapping (L.Michel proposal) attempt for annotating tables containing more than 1 transformation**



# Conclusion

- 1 Most of the problems for polynomial distortions solved.**
  - 2 Why do we have Toperation/Tfunction separation (One Operation contains only one function and nothing else)**
  - 3 ° JSON (Or YAML) « Official » serialisation to be considered to go with individual images (FITEX extension?)**
  - 4 ) VODML-lite mapping feasible**
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